THE ANTI-SYMMETRIC ORTHO-SYMMETRIC SOLUTIONS OF THE MATRIX EQUATION $A^TXA = D^*$

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Abstract. In this paper, the following problems are discussed.

Problem I. Given matrices $A \in \mathbb{R}^{n \times m}$ and $D \in \mathbb{R}^{m \times m}$, find $X \in ASR^n_P$ such that

$$A^TXA = D,$$

where

$$ASR^n_P = \{X \in ASR^{n \times n}|PX \in SR^{n \times n} \text{ for given } P \in OR^{n \times n} \text{ satisfying } P^T = P\}.$$  

Problem II. Given a matrix $\tilde{X} \in \mathbb{R}^{n \times n}$, find $\hat{X} \in SE$ such that

$$\|\tilde{X} - \hat{X}\| = \inf_{X \in SE} \|\tilde{X} - X\|,$$

where $\|\cdot\|$ is the Frobenius norm, and $SE$ is the solution set of Problem I.

Expressions for the general solution of Problem I are derived. Necessary and sufficient conditions for the solvability of Problem I are provided. For Problem II, an expression for the solution is given as well.

Key words. Anti-symmetric ortho-symmetric matrix, Matrix equation, Matrix nearness problem, Optimal approximation, Least-square solutions.

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