SPECTRA OF WEIGHTED COMPOUND GRAPHS OF
GENERALIZED BETHE TREES∗

OSCAR ROJO† AND LUIS MEDINA†

Abstract. A generalized Bethe tree is a rooted tree in which vertices at the same distance from the root have the same degree. Let \( G_m \) be a connected weighted graph on \( m \) vertices. Let \( \{B_i : 1 \leq i \leq m\} \) be a set of trees such that, for \( i = 1, 2, \ldots, m \),

(i) \( B_i \) is a generalized Bethe tree of \( k_i \) levels,

(ii) the vertices of \( B_i \) at the level \( j \) have degree \( d_{i,k_i-j+1} \) for \( j = 1, 2, \ldots, k_i \), and

(iii) the edges of \( B_i \) joining the vertices at the level \( j \) with the vertices at the level \( (j+1) \) have weight \( w_{i,k_i-j} \) for \( j = 1, 2, \ldots, k_i - 1 \).

Let \( G_m \{B_i : 1 \leq i \leq m\} \) be the graph obtained from \( G_m \) and the trees \( B_1, B_2, \ldots, B_m \) by identifying the root vertex of \( B_i \) with the \( i \)th vertex of \( G_m \). A complete characterization is given of the eigenvalues of the Laplacian and adjacency matrices of \( G_m \{B_i : 1 \leq i \leq m\} \) together with results about their multiplicities. Finally, these results are applied to the particular case \( B_1 = B_2 = \cdots = B_m \).

Key words. Weighted graph, Generalized Bethe tree, Laplacian matrix, Adjacency matrix, Spectral radius, Algebraic connectivity.

AMS subject classifications. 5C50, 15A48.

∗Received by the editors June 30, 2008. Accepted for publication December 30, 2008. Handling Editor: Michael Neumann.
†Department of Mathematics, Universidad Católica del Norte, Antofagasta, Chile (orojo@ucn.cl). The work of the first author was supported by Project Fondecyt 1070537, Chile. The work of the second author was supported by Project Mecesup UCN0202, Chile.