THE NUMERICAL RADIUS OF A WEIGHTED SHIFT OPERATOR WITH GEOMETRIC WEIGHTS

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Abstract. Let $T$ be a weighted shift operator on the Hilbert space $\ell^2(N)$ with geometric weights. Then the numerical range of $T$ is a closed disk about the origin, and its numerical radius is determined in terms of the reciprocal of the minimum positive root of a hypergeometric function. This function is related to two Rogers-Ramanujan identities.

Key words. Numerical radius, Weighted shift operator, Rogers-Ramanujan identities.

AMS subject classifications. 47A12, 47B37, 33D15.

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