ZERO-NONZERO PATTERNS FOR NILPOTENT MATRICES OVER FINITE FIELDS

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Abstract. Fix a field \(F\). A zero-nonzero pattern \(\mathcal{A}\) is said to be potentially nilpotent over \(F\) if there exists a matrix with entries in \(F\) with zero-nonzero pattern \(\mathcal{A}\) that allows nilpotence. In this paper an investigation is initiated into which zero-nonzero patterns are potentially nilpotent over \(F\) with a special emphasis on the case that \(F = \mathbb{Z}_p\) is a finite field. A necessary condition on \(F\) is observed for a pattern to be potentially nilpotent when the associated digraph has \(m\) loops but no small \(k\)-cycles, \(2 \leq k \leq m - 1\). As part of this investigation, methods are developed, using the tools of algebraic geometry and commutative algebra, to eliminate zero-nonzero patterns \(\mathcal{A}\) as being potentially nilpotent over any field \(F\). These techniques are then used to classify all irreducible zero-nonzero patterns of order two and three that are potentially nilpotent over \(\mathbb{Z}_p\) for each prime \(p\).

Key words. Zero-nonzero patterns, Nilpotent, Ideal saturation, Gröbner basis, Finite fields.

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