SIGN PATTERNS THAT REQUIRE EVENTUAL POSITIVITY OR REQUIRE EVENTUAL NONNEGATIVITY∗

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Abstract. It is shown that a square sign pattern \( A \) requires eventual positivity if and only if it is nonnegative and primitive. Let the set of vertices in the digraph of \( A \) that have access to a vertex \( s \) be denoted by \( \text{In}(s) \) and the set of vertices to which \( t \) has access denoted by \( \text{Out}(t) \). It is shown that \( A = [\alpha_{ij}] \) requires eventual nonnegativity if and only if for every \( s, t \) such that \( \alpha_{st} = -\), the two principal submatrices of \( A \) indexed by \( \text{In}(s) \) and \( \text{Out}(t) \) require nilpotence. It is shown that \( A \) requires eventual exponential positivity if and only if it requires exponential positivity, i.e., \( A \) is irreducible and its off-diagonal entries are nonnegative.

Key words. Eventually nonnegative matrix, Eventually positive matrix, Eventually exponentially positive matrix, Exponentially positive matrix, Sign pattern, Perron-Frobenius.

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