THE $q$-NUMERICAL RANGE OF $3 \times 3$ TRIDIAGONAL MATRICES

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Abstract. For $0 \leq q \leq 1$, we examine the $q$-numerical ranges of $3 \times 3$ tridiagonal matrices $A(b)$ that interpolate between the circular range $W_0(A(b))$ and the elliptical range $W_1(A(b))$ as $q$ varies from 0 to 1. We show that for $q \leq (1 - b^2)/(2(1 + b^2))$, $W_q(A(b))$ is a circular disc centered at the origin with radius $(1 + b^2)^{1/2}$, but $W_{4/5}(A(2))$ is not even an elliptical disc.

Key words. Tridiagonal matrix, Davis-Wielandt shell, $q$-numerical range.

AMS subject classifications. 15A60.