REACHABILITY MATRICES AND CYCLIC MATRICES*

AUGUSTO FERRANTE† AND HARALD K. WIMMER‡

Abstract. We study reachability matrices \( R(A, b) = [b, Ab, \ldots, A^{n-1}b] \), where \( A \) is an \( n \times n \) matrix over a field \( K \) and \( b \) is in \( K^n \). We characterize those matrices that are reachability matrices for some pair \((A, b)\). In the case of a cyclic matrix \( A \) and an \( n \)-vector of indeterminates \( x \), we derive a factorization of the polynomial \( \det(R(A, x)) \).

Key words. Reachability matrix, Krylov matrix, cyclic matrix, nonderogatory matrix, companion matrix, Vandermonde matrix, Hautus test.

AMS subject classifications. 15A03, 15A15, 93B05.

*Received by the editors September 23, 2009. Accepted for publication February 11, 2010. Handling Editor: Daniel Szyld.
†Dipartimento di Ingegneria dell’Informazione, Università di Padova, I-35131 Padova, Italy (augusto@dei.unipd.it).
‡Mathematisches Institut, Universität Würzburg, D-97074 Würzburg, Germany (wimmer@mathematik.uni-wuerzburg.de).