Quasi-stationary Distributions: A Bibliography

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Last updated 26th May 2011

Abstract

Quasi-stationary distributions have been used to model the long-term behaviour of stochastic systems which in some sense terminate, but appear to be stationary over any reasonable time scale. The idea can be traced back to the work of the Russian Mathematician A.M. Yaglom, who showed that the limiting conditional distribution of the number in the \( n \)th generation of the Galton Watson branching process always exists in the subcritical case (see Yaglom [416]). But, it was not until the early sixties, and largely stimulated by the remarkable work of Vere-Jones [405], and later Kingman [199], Darroch and Seneta [90], Seneta and Vere-Jones [353], and Darroch and Seneta [91], that a general theory was annunciated. Since then, quasi-stationary distributions have appeared in a variety of diverse contexts, including chemical reaction kinetics, reliability theory, genetics, epidemics, ecology and telecommunications, and this work has stimulated further developments in the theory. Modern key papers in the area are Ferrari, Kesten, Martínez and Picco [107] and Kesten [179].

I present here a bibliography of work on quasi-stationary distributions. This includes work on quasi-stationary distributions per se (stationary conditional distributions), limiting conditional distributions (often called quasi-stationary distributions, and also called Yaglom limits and quasi-limiting distributions), the companion topics of geometric and exponential ergodicity, \( R \)-classification of states and \( R \)-invariant measures (et cetera), ratio limit theorems, analysis of processes conditioned to stay within a given region (particularly weak convengence of those processes), and papers dealing with diffusion approximations which specifically describe quasi stationarity of evanescent processes.

Published work is cited under various headings. Several works appear under more than one heading. The final section lists the same works in chronological order.

Whilst I do not claim that the bibliography is exhaustive, I do hope that it includes most of the work published on quasi-stationary distributions. I welcome additions and corrections. I would particularly like to hear about Ph.D. theses in the area (they are very difficult to trace). Please e-mail me at pkp@maths.uq.edu.au. This bibliography is maintained at

1 Textbooks

As yet there is no textbook specifically devoted to quasi-stationary distributions. However, the following texts contained sections devoted to quasi-stationary distributions or the idea of a limiting conditional distribution.

Anderson [8]
Bartlett [30]
Kijima [189]
May [249]
Seneta [351]

2 Ph.D. theses

Breyer [49]
Day [92]
Hart [144]
Parsons [290]
Sirl [363]
Walker [410]

3 General theory

This section on the general theory of quasi-stationary distributions also including papers on the related topics of geometric and exponential ergodicity, classifications of states, and ratio limit theorems.

3.1 Ratio limit theory

Cheong [71]
Cohn [78]
Collet, Martínez and San Martín [80]
Doney [97]
Enderle and Hering [101]
Ferrari, Kesten, Martínez and Picco [107]
Foguel [117]
Foguel and Lin [118]
Gerl [122]
Giné, Koltchinskii and Wellner [128]
Handelman [141]
Hou [154]
Isaac [159]
Isaac [160]
Kersting [176]
Kesten [177]
3.2 Discrete-time Markov chains

Al-Eideh [2]
Al-Eideh [3]
Al-Eideh and Al-Towaiq [4]
Al-Towaiq and Al-Eideh [5]
Al-Towaiq and Al-Eideh [6]
Brown [54]
Buckley and Pollett [56]
Buiculescu [58]
Callaert [61]
Coolen-Schrijner and Pollett [83]
Coolen-Schrijner and van Doorn [84]
Darroch and Seneta [90]
Isaacson [161]
3.3 Continuous-time Markov chains

Abate and Whitt [1]
Anisimov and Pushkin [10]
Artalejo and Lopez-Herrero [15]
Breyer and Roberts [52]
Buiculescu [57]
Buiculescu [60]
Callaert [61]
Chen and Strook [69]
Coolen-Schrijner, Hart and Pollett [82]
Darlington and Pollett [89]
Darroch and Seneta [91]
Elmes, Pollett and Walker [99]
Elmes, Pollett and Walker [100]
Ferrari, Martínez and Picco [112]
Ferrari, Kesten, Martínez and Picco [107]
Flaspohler [115]
Gray, Pollett and Zhang [136]
Hart and Pollett [147]
Hart and Pollett [146]
Hart and Pollett [148]
Hart, Martínez and San Martín [145]
Jacka and Roberts [163]
Kijima [187]
3.4 Semi-Markov and Markov-renewal processes

Arjas and Nummelin [12]
Arjas, Nummelin and Tweedie [13]
Cheong [70]
Cheong [71]
Cheong [73]
3.5 Markov processes on a general state space

Arjas, Nummelin and Tweedie [13]
Bebbington, Pollett and Zheng [35]
Bertoin [40]
Breyer and Roberts [52]
Enderle and Hering [101]
Folkman and Port [119]
Glover [129]
Isaac [160]
Lambert [220]
Lin [236]
Klebaner, Lazar and Zeitouni [203]
Nummelin and Arjas [270]
Nummelin and Tweedie [271]
Orey [276]
Pollard and Tweedie [303]
Pollard and Tweedie [304]
Roberts [338]
Tuominen and Tweedie [379]
Tuominen and Tweedie [381]
Tweedie [389]
Tweedie [387]
Tweedie [388]
Tweedie [391]

3.6 Dynamical systems

Khasminskii, Yin and Zhang [180]
Li, Yin, Yin and Zhang [232]
Klebaner and Lazar [202]

3.7 Miscellaneous

Asselah and Castell [16]
Asselah and Dai Pra [17]
4 Diffusion approximations

Barbour [23]
Barbour [24]
Barbour [25]
Barbour [26]
Barbour [27]
Barbour [28]
Kurtz [207]
Kurtz [208]
Kurtz [210]
Kurtz [211]
McNeill and Schach [250]
5 Special processes

5.1 Cellular automata

Atman and Dickman [19]
Ferrari, Kesten and Martínez [106]
Martínez [244]

5.2 Birth-death processes

Artalejo and Lopez-Herrero [15]
Callaert and Keilson [63]
Callaert and Keilson [64]
Callaert [62]
Cavender [66]
Chan [67]
Clancy and Pollett [76]
Coolen-Schrijner and van Doorn [85]
Ferrari, Martínez and Picco [111]
Good [132]
Keilson and Ramaswamy [169]
Keilson and Ramaswamy [170]
Kesten [178]
Kijima [184]
Kijima [182]
Kijima, Nair, Pollett and van Doorn [194]
Kijima and Seneta [195]
Lambert [220]
Mandl [243]
Martínez [244]
Martínez and Vares [248]
Parthasarathy, Lenin, Schoutens and van Assche [292]
Roberts and Jacka [339]
Roberts, Jacka and Pollett [340]
Schoutens [345]
Schrijner and van Doorn [346]
Van Doorn [394]
Van Doorn [396]
Van Doorn and Schrijner [401]
Van Doorn and Schrijner [400]

5.3 Branching processes

Bagley [22]
5.4 Brownian motion

Awad and Glynn [21]
Collet, Martínez and San Martín [80]
Ferrari, Martínez and San Martín [113]
Garbit [120]
Housworth [155]
Klass and Pitman [201]
Martínez and San Martín [247]
Martínez Picco and San Martín [246]
Salminen [342]
Serlet [356]

5.5 Catastrophe processes

Pakes [281]
Pakes and Pollett [287]

5.6 Diffusions and related processes

Cattiaux, Collet, Lambert, Martínez, Méléard and San Martín [65]
Collet, Martínez and San Martín [79]
Kennedy [175]
Kao [167]
Lambert [218]
Pinsky [299]
5.7 Quasi-birth-death processes

Bean, Bright, Latouche, Pearce, Pollett and Taylor [31]
Bean, Pollett and Taylor [32]
Bean, Pollett and Taylor [33]
Bean, Pollett and Taylor [34]
Pearce [294]

5.8 Queues and related models

Awad and Glynn [21]
Boucherie [46]
Ferrari and Lopes Garcia [108]
Kennedy [175]
Kibkalo [181]
Kijima [185]
Kijima and Makimoto [191]
Kijima and Makimoto [192]
Kijima and Makimoto [193]
Kijima and Makimoto [190]
Kyprianou [214]
Kyprianou [215]
Kyprianou [216]
Li [228]
Li and Zhao [229]
Li and Zhao [230]
Makimoto [240]
Topolski [377]
Tuominen and Tweedie [380]
Van Doorn and Regterschot [398]

5.9 Random walks

Bertoin and Doney [41]
Bertoin and Doney [42]
Bertoin and Doney [43]
Bolthausen [45]
Daley [86]
Doney [95]
Doney [96]
Doney [97]
Iglehart [158]
Iglehart [157]
Kao [167]
Keener [168]
Pakes [279]
Ritter [337]
Seneta [347]
Shimura [357]
Stadje [364]
Szubarga and Szynal [373]
Szubarga and Szynal [372]
Szubarga and Szynal [374]
Zhao and Ying [420]

6 Computational methods

Bebbington [36]
Bebbington [37]
Bebbington and Stewart [38]
Boucherie and van Doorn [47]
Pollett [314]
Pollett and Roberts [328]
Pollett and Stewart [329]

7 Truncation methods

Most of the papers listed in this section concern the evaluation of stationary distributions, but I have included here work which might easily be adapted to handle quasi-stationary distributions.

Breyer and Hart [50]
Breyer and Hart [51]
Gibson and Seneta [125]
Gibson and Seneta [124]
Hart and Tweedie [149]
Pearce and Shin [293]
Seneta [349]
Seneta [350]
Tweedie [384]
8 Applications

8.1 Biology and ecology

Artalejo and Lopez-Herrero [15]
Bartlett [29]
Becker [39]
Buckley and Pollett [55]
Buckley and Pollett [56]
Day and Possingham [93]
Ferrari and Marić [109]
Gilpin and Hanski [126]
Gilpin and Taylor [127]
Gosselin [133]
Gosselin [134]
Grasman [135]
Gyllenberg and Silvestrov [138]
Hanson and Tuckwell [142]
Hanson and Tuckwell [143]
Högnäs [152]
Holling [153]
Klein [204]
Kukhtin, Kuzmenko and Shramko [206]
Lambert [220]
Lambert [221]
Mech [251]
Pakes, Trajstman and Brockwell [288]
Pollett [306]
Pollett [315]
Pollett [318]
Pollett [319]
Pollett [321]
Pollett [320]
Pollett [325]
Pollett [323]
Scheffer [344]
Trajstman [378]

8.2 Chemical kinetics

Dambrine and Moreau [87]
8.3 Epidemics

Andersson and Britton [9]
Artalejo, Economou and Lopez-Herrero [14]
Artalejo and Lopez-Herrero [15]
Clancy, O’Neill and Pollett [75]
Clancy and Pollett [76]
Hernández-Suárez and Castillo-Chavez [151]
Nåsell [260]
Nåsell [261]
Nåsell [262]
Nåsell [263]
Nåsell [264]
Nåsell [265]
Ovaskainen [278]
Sani, Kroese and Pollett [343]

8.4 Genetics

Lambert [220]
Lambert [221]
Ewens [103]
Ewens [104]
Kendall [174]
Seneta [348]

8.5 Reliability

Cocozza-Thivent and Roussignol [77]
Kalpakam and Shahul Hameed [165]
Pijnenburg and Ravichandran [297]
Pijnenburg, Ravichandran and Regterschot [298]
8.6 Telecommunications

Gibbens, Hunt and Kelly [123]
Kelly [172]
Kelly [173]
Pollett [311]
Pollett [310]
Pollett [313]
Ziedins [421]

8.7 Medicine

Chan, Pollett and Weinstein [68]

9 Significant related material

Brockwell, Gani and Resnick [53]
Kelly [171]
Kijima and van Doorn [196]
Ledermann and Reuter [223]
Li and Li [233]
Liu, Zhang and Zhao [238]
Pakes [284]
Pakes [285]
Peng [295]
Pollett [326]
Pollett [327]
Pruitt [334]
Sirl, Zhang and Pollett [362]
Van Doorn and Schrijner [399]
Van Doorn [393]
Van Doorn [395]
10 Chronological order

1947
Yaglom [416]

1951
Scheffer [344]

1954
Ledermann and Reuter [223]

1957
Bartlett [29]
Zolotarev [422]

1958
Waugh [412]

1960
Bartlett [30]
Mandl [242]

1962
Vere-Jones [405]

1963
Albert [7]
Ewens [103]
Kingman [199]
Kingman [198]

1964
Ewens [104]
Kingman and Orey [200]
Mandl [243]
Pruiit [334]

1965
Darroch and Seneta [90]
Port [333]
Pruiit [335]

1966
Folkman and Port [119]
Kendall [174]
Mech [251]
Seneta [347]
Seneta [348]
Seneta and Vere-Jones [353]
Stone [367]

1967
Cheong [70]
Darroch and Seneta [91]
Heathcote, Seneta and Vere-Jones [150]
Isaac [159]
Orey [276]
Papangelou [289]
Seneta [349]
Stone [368]
Vere-Jones [406]

1968
Cheong [71]
Good [132]
Klein [204]
Seneta and Vere-Jones [354]
Seneta [350]
Teugels [376]
Teugels [375]
Vere-Jones [407]

1969
Daley [86]
Foguel [117]
Vere-Jones [408]

1970
Becker [39]
Cheong [73]
Cheong [72]
Kesten [178]
Kesten [177]
Kurtz [207]
Lin [234]
Levitan [226]

1971
Kurtz [208]
Kyprianou [214]
Levitan [227]
Tweedie [384]

1972
Buiculescu [57]
Cheong [74]
Flaspohler and Holmes [116]
Foguel and Lin [118]
Isaac [159]
Kesten [177]
Kingman and Orey [200]
Levitan [226]
Levitan [227]
Lin [234]
Lin [235]
Papangelou [289]
Port [333]
Pruitt [335]
Stone [367]
Stone [368]

1973
Buiculescu [58]
Callaert and Keilson [63]
Callaert and Keilson [64]
Holling [153]
Kurtz and Wainger [212]
Lamb [217]
McNeil and Schach [250]
Pakes [279]
Tweedie [385]

1974
Barbour [23]
Barbour [24]
Callaert [61]
Callaert [62]
Colm [78]
Flaspohler [115]
Gerl [122]
Iglehart [158]
Iglehart [157]
Kennedy [175]
Kennedy [175]
Kersting [176]
May [249]
Tweedie [389]
Tweedie [387]
Tweedie [388]
Tweedie [386]

1975
Buiculescu [59]
Malek-Mansour and Nicolis [241]
Narimanjan [259]
Pakes [280]
Pollard and Tweedie [303]
Tweedie [390]
Yong [418]

1976
Barbour [25]
Bolthausen [45]
Kurtz [210]
Lin [236]
Mucci [256]
Nummelin [266]
Nummelin and Arjas [270]
Pollard and Tweedie [304]

1977
Arjas and Nummelin [12]
Green [137]
Móricz [254]
Nummelin [267]
Oman [272]
Oppenheim, Schuler and Weiss [275]

1978
Cavender [66]
Evans [102]
Hanson and Tuckwell [142]
Kao [167]
Kao [167]
Kurtz [211]
Nummelin [268]
Nummelin and Tweedie [271]
Oman [273]
Turner and Malek-Mansour [383]

1979
Barbour [26]
Isaacson [161]
Nummelin [269]
Pakes, Trajstman and Brockwell [288]
Tuominen and Tweedie [379]
Tuominen and Tweedie [380]

1980
Arjas, Nummelin and Tweedie [13]
Arjas, Nummelin and Tweedie [13]
Barbour [27]
Barbour [28]
Móricz [255]
Serfling [355]
Sur [370]
Van Doorn [393]

1981

Dambrine and Moreau [87]
Dambrine and Moreau [88]
Hanson and Tuckwell [143]
Ritter [337]
Seneta [351]
Trajstman [378]
Tweedie [391]

1982

Bagley [22]
Brockwell, Gani and Resnick [53]
Enderle and Hering [101]
Zuparov and Mamadaliev [423]

1983

Chen and Strook [69]
Doney [95]
Ezhov and Reshetnyak [105]
Isaac [160]
Kalpakam and Shahul Hameed [165]
Kelly [171]
ˇSur [371]
Turkman [382]

1984

Keilson and Ramaswamy [169]

1985

Anisimov and Pushkin [10]
Doney [96]
Kelly [172]
Kibkalo [181]
Parsons [290]
Pinsky [299]
Seneta and Tweedie [352]
Szubarga and Szynal [373]
Szubarga and Szynal [372]
Szubarga and Szynal [374]
Van Doorn [394]

1986
Glover [129]
Keilson and Ramaswamy [170]
Pollak and Siegmund [301]
Pollett [305]

1987
Gibson and Seneta [125]
Gibson and Seneta [124]
Kelly [173]
Pakes [281]
Parsons and Pollett [291]
Pollett [306]
Rootzén [341]
Shur [358]
Van Doorn [395]
Ziedins [421]

1988
Jacka and Roberts [162]
Pollett [308]
Pollett [307]
Topolski [377]
Van Doorn and Regterschot [398]

1989
Abate and Whitt [1]
Buiculescu [60]
Pakes and Pollett [287]
Pollett [309]

1990
Al-Eideh and Al-Towaiq [4]
Gibbens, Hunt and Kelly [123]
Pijnenburg and Ravichandran [297]
Pijnenburg, Ravichandran and Regterschot [298]
Pollett and Roberts [328]

1991
Al-Towaiq and Al-Eideh [5]
Anderson [8]
Brown [54]
Ferrari, Martínez and Picco [111]
Gilpin and Hanski [126]
Kijima and Seneta [195]
Näsell [260]
Pollett [312]
Pollett [311]
Pollett [310]
Roberts [338]
Shimura [357]
Van Doorn [396]

1992

Ferrari, Martínez and Picco [112]
Keener [168]
Kijima [184]
Kijima [183]
Kijima and Makimoto [191]
Pollett [313]
Pollett and Vassallo [330]
Pollett and Vere-Jones [331]
Salminen [342]

1993

Hou [154]
Kijima [187]
Kijima [186]
Kijima [185]
Kijima [182]
Klass and Pitman [201]
Nair and Pollett [258]
Nair and Pollett [257]
Makimoto [240]
Martínez [244]
Pakes [282]
Pakes [283]
Pinsky [300]
Pollett [316]
Pollett [314]
Pollett [315]
Van Doorn and Schrijner [399]

1994

Al-Eideh [2]
Al-Towaiq and Al-Eideh [6]
Bertoin and Doney [41]
Bertoin and Doney [42]
Bertoin and Doney [43]
Ferrari and Martínez [110]
Gilpin and Taylor [127]
Gyllenberg and Silvestrov [138]
Housworth [155]
Kijima and Makimoto [192]
Kimmel [197]
Ledoux, Rubino and Sericola [225]
Ledoux, Rubino and Sericola [225]
Martínez and San Martín [247]
Pollett and Stewart [329]
Roberts and Jacka [339]
Tuominen and Tweedie [381]

1995

Al-Eideh [3]
Bebbington [36]
Bebbington, Pollett and Zheng [35]
Collet, Martínez and San Martín [79]
Day [92]
Day and Possingham [93]
Dykman, Horita and Ross [98]
Ferrari, Kesten, Martínez and Picco [107]
Jacka and Roberts [163]
Kesten [179]
Kijima [188]
Kijima and Makimoto [193]
Kijima and van Doorn [196]
Ledoux [224]
Martínez and Vares [248]
Pakes [284]
Pakes [286]
Pollard [302]
Pollett [317]
Van Doorn and Schrijner [401]
Van Doorn and Schrijner [400]
Wang and Wang [411]

1996

Bean, Pollett and Taylor [32]
Bebbington and Stewart [38]
Breyer and Hart [50]
Cocozza-Thivent and Roussignol [77]
Collet, Martínez and Schmitt [81]
Elmes, Pollett and Walker [99]
Ferrari, Kesten and Martínez [106]
Grasman [135]
Hart and Pollett [147]
Hart and Pollett [146]
Jacka and Roberts [164]
Khasminskii, Yin and Zhang [180]
Klebaner and Lazar [202]
Nåsell [261]
Peng [295]
Pollett [318]
Serlet [356]
Van Doorn and Schrijner [403]
Van Doorn and Schrijner [402]

1997

Bean, Bright, Latouche, Pearce, Pollett and Taylor [31]
Bebbington [37]
Bertoin [40]
Boucherie [46]
Breyer [49]
Ferrari, Martínez and San Martín [113]
Hart [144]
Höggnäs, [152]
Kijima [189]
Kijima, Nair, Pollett and van Doorn [194]
Kukhtin, Kuzmenko and Shramko [206]
Pakes [285]
Petersen and Schmidt [296]
Pollett [319]
Roberts, Jacka and Pollett [340]
Schrijner and van Doorn [346]
Stadje [364]

1998

Bean, Pollett and Taylor [33]
Boucherie and van Doorn [47]
Chan [67]
Doney [97]
Ferrari and Lopes Garcia [108]
Kang and Klotz [166]
Klebaner, Lazar and Zeitouni [203]
Knoth [205]
Martínez Picco and San Martín [246]
Násell [262]
Parthasarathy, Lenin, Schoutens and van Assche [292]
Pearce [294]
Pearce and Shin [293]
Tweedie [392]
Walker [410]
Walker [409]

1999

Breyer and Roberts [52]
Collet, Martínez and San Martín [80]
Coolen-Schrijner and Pollett [83]
Fierro, Martínez and San Martín [114]
Gyllenberg and Silvestrov [139]
Handelman [141]
Kijima and Makimoto [190]
Li, Yin, Yin and Zhang [232]
Näsell [263]
Näsell [264]
Pollett [322]
Pollett [321]
Pollett [320]
Ramanan and Zeitouni [336]
Strunkov [369]

2000

Andersson and Britton [9]
Bean, Pollett and Taylor [34]
Coolen-Schrijner, Hart and Pollett [82]
Darlington and Pollett [89]
Elmes, Pollett and Walker [100]
Geiger [121]
Gyllenberg and Silvestrov [140]
Hart and Pollett [148]
Lambert [218]
Lladser and San Martín [239]
Moler, Plo and San Miguel [253]
Oshima [277]
Schoutens [345]

2001

Asselah and Dai Pra [17]
Breyer, Roberts and Rosenthal [48]
Clancy, O’Neill and Pollett [75]
Glynn and Thorisson [130]
Gosselin [134]
Lasserre and Pearce [222]
Li and Xiao [231]
Mei and Lin [252]
Ovaskainen [278]
Pollett [325]
Pollett [323]
Wu [413]

2002

Asselah and Ferrari [18]
Atman and Dickman [19]
Glynn and Thorisson [131]
Li and Zhao [229]
Lin, Zhang and Hou [237]
Pollett [324]
Shur [359]
Zhao and Jin [419]
Dickman and Vidigal [94]

2003
Asselah and Castell [16]
Clancy and Pollett [76]
Giné, Koltchinskii and Wellner [128]
Hart, Martínez and San Martín [145]
Li and Zhao [230]
Wu [414]

2004
Huisinga, Meyn and Schütte [156]
Pollett and Zhang [332]
Shur [360]
Steinsaltz Evans [365]
Vatutin and Dyakonova [404]
Wu [415]

2005
Gray, Pollett and Zhang [136]
Zhao and Ying [420]
Hart and Tweedie [149]
Näsell [265]

2006
Coolen-Schrijner and van Doorn [84]
Coolen-Schrijner and van Doorn [85]
Kyprianou and Palmowski [213]

2007
Ferrari and Marić [109]
Lambert [219]
O’Neill [274]
Pollett [326]
Sani, Kroese and Pollett [343]
Silvestrov [361]
Sirl, Zhang and Pollett [362]
Steinsaltz and Evans [366]

2008
Lambert [220]
Liu, Zhang and Zhao [238]
Martínez [245]
Pollett [327]
Ye [417]

2009
Arguin and Aizenman [11]
Awad and Glynn [21]
Buckley and Pollett [55]
Cattiaux, Collet, Lambert, Martínez, Mélard and San Martín [65]
Chan, Pollett and Weinstein [68]
Garbit [120]
Li and Li [233]
Van Doorn and Pollett [397]

2010

Artalejo, Economou and Lopez-Herrero [14]
Artalejo and Lopez-Herrero [15]
Avrachenkov, Borkar and Nemirovsky [20]
Buckley and Pollett [56]
Hernández-Suárez and Castillo-Chavez [151]
Lambert [221]
Li [228]
References


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